A weather balloon with mass \( m \) and initial volume \( V_0 \) ascends in an isothermal atmosphere. Its envelope is loose up to the achievement of the maximal volume \( V_1 \).

\[ p_0 = 10^5 \, \text{N/m}^2 \quad \rho_0 = 1.27 \, \text{kg/m}^3 \quad m = 2.5 \, \text{kg} \quad V_0 = 2.8 \, \text{m}^3 \quad V_1 = 10 \, \text{m}^3 \]

\[ R = 287 \, \text{Nm/kgK} \quad g = 10 \, \text{m/s}^2 \]

a) What is the necessary force to hold down the balloon before launch?

b) In what altitude the balloon reaches its maximum volume \( V_1 \)?
c) What ceiling reaches the balloon?
a) before launch

\[ \Sigma F_z = 0 = F_A - F_G - F_N - F_H \]

\[ F_H = F_A - (F_N + F_G) = \]

\[ = \rho_L(z = 0)V_0 g - mg = \]

\[ = (\rho_0 V_0 - m)g = 10.6 \text{ N} \]
b) \[ z \text{ for } V = V_1 \]

perfectly loose for \( V < V_1 \)

the envelope can change its volume

\[
m_G = \text{const} = \rho G V = \frac{p_G}{R_G T_G} V
\]

\[ p_i = p_a \]

The movement is quite slow: \[ \rightarrow T_i = T_a \]

Assumption: isothermal atmosphere \[ \rightarrow \text{scale height relation} \]

\[
V = \frac{m_G R_G T_G}{p_G} \sim \frac{1}{p_G} = \frac{1}{p_L}
\]
\[ z = 0 \rightarrow V = V_0 \]

\[ V(z) = V_0 e^{g z / RLT_0} \]

\[ V_1 = V(z = z_1) = V_0 e^{g z_1 / RLT_0} \]

\[ z_1 = \ln \left( \frac{V_1}{V_0} \right) \frac{RLT_0}{g} \]

\[ \frac{p_0}{\rho_0} = RLT_0 \]

\[ z_1 = \frac{p_0}{\rho_0 g} \ln \left( \frac{V_1}{V_0} \right) = 10.0 \text{ km} \]
The lift force onto a loose balloon is constant. $(T_L = T_G, g = \text{const})$

\[
F_A(z \leq z_1) = \rho_0 V_0 g = \rho_L(z_1)V_1 g
\]

\[
F_A(z > z_1) = \rho_L(z)V_1 g
\]

\[
F_A(z > z_1) = F_A(z \leq z_1) \cdot \frac{\rho_L(z)}{\rho_L(z_1)} = \rho_L(z) \cdot e^{-\frac{g(z-z_1)}{R_LT_0}}
\]
ceiling: \( \sum F_z = 0 \rightarrow mg = F_A \)

\[
= mg - \rho(z_{max})V_1g
\]

\[
\rightarrow \rho_0 e^{-\frac{g z_{max}}{R_L T_0}} = \frac{m}{V_1}
\]

\[
z_{max} = \frac{R_L T_0}{g} \ln \frac{V_1 \rho_0}{m} \frac{\rho_0}{\rho g} \ln \frac{V_1 \rho_0}{m} =
\]

\[
= 12.8 \text{ km}
\]